Homework 7

Instructions: The file must be in pdf extension. Show neat and complete work and make sure that your scan is legible. Label your solutions and make sure they are in increasing order.

You are allowed to use the Laplace transform table but not everything follows directly from the table. For partial fractions guide you can follow this link:`https://tutorial.math.lamar.edu/classes/calcii/partialfractions.aspx`

1. Find the Laplace transform of $1 + t + t^2$.
2. Find the Laplace transform of $(t + 1)^3$.
3. Find the Laplace transform of $1 - t^7 + \cos(\pi t) + e^{sin(3)t}$.
4. Find the Laplace transform of $sin(\omega(t - a))$.
5. Find the Laplace transform of te^{-t}.
6. Find the Laplace transform of

$$f(x) = \begin{cases} t & t \geq 1 \\ 0 & t < 1 \end{cases}$$

7. Find the inverse Laplace transform of $\frac{4}{s^2-9}$.
8. Prove the shifting property of the Laplace transform:

$$\mathcal{L}\{e^{-at} f(t)\} = F(s + a)$$

9. Find the inverse Laplace transform of $\frac{s+2}{(s+2)^2+\omega^2}$.
10. Find the inverse Laplace transform of $\frac{1}{(s-1)^2(s+1)}$.

11. Find the inverse Laplace transform of $\frac{8}{s^3(s+2)}$.

12. The Heaviside function is defined as

$$u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

Find the Laplace transform of the shifted Heaviside function $u(t-a)$.

13.

$$f(t) = \begin{cases} 0 & t < 0 \\ t^2 & 0 \leq t \leq 1 \\ t & t > 1 \end{cases}$$

Using the Heaviside function write down the piecewise function $f(t)$.

14. Prove the shifting property of the Heaviside function:

$$\mathcal{L}\{f(t-a)u(t-a)\} = e^{-as}\mathcal{L}\{f(t)\}\}$$

15. Find

$$\mathcal{L}^{-1}\Big\{\frac{e^{-3s}}{s(s^2+1)}\Big\}$$

(Hint: First find inverse Laplace of $\frac{1}{s(s^2+1)}$ and then use problem 14.